Anisotropic Density Estimation for Photon Mapping

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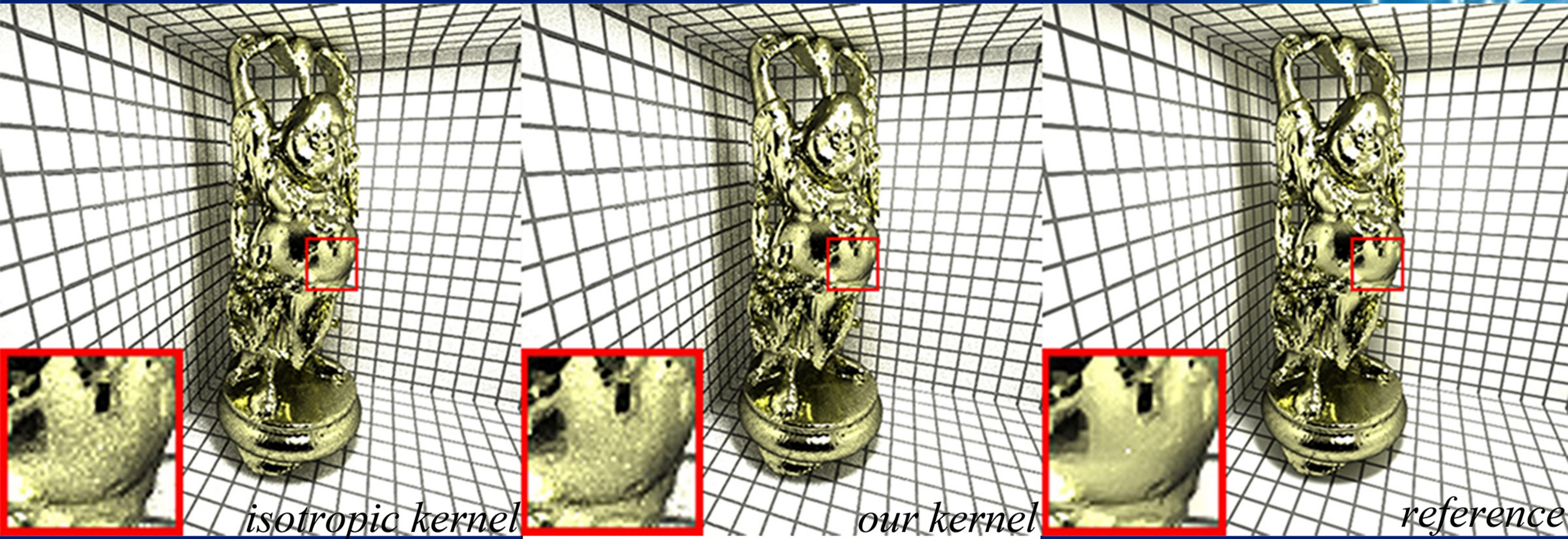

1. Introduction

Photon Mapping is a widely used technique for global illumination rendering. In the density estimation step of photon mapping, the indirect radiance at a shading point is estimated through a filtering from nearby photons, where an *isotropic filtering kernel* is usually used. However, using isotropic kernel is not the optimal choice for cases when eye paths intersect with surfaces of anisotropic BRDFs.

We propose an *anisotropic filtering kernel* for density estimation to handle such anisotropic eye paths. The anisotropic filtering kernel is derived from the recently introduced anisotropic spherical Gaussian [XSD*13] representation of BRDFs. Compared to conventional photon mapping, our method is able to reduce rendering errors with only negligible additional costs in rendering scenes containing anisotropic BRDFs.

2. Technical Approach

As shown in Figure 1, considering a typical anisotropic eye path which starts from viewpoint to a point p_0 on an anisotropic surface, then reflected to the density estimation point p_1 on a diffuse surface, the anisotropic filtering kernel is computed and applied through 4 steps:

1.Spherical Warping.

We use ASGs to represent the anisotropic BRDF at p_0. ASG based BRDF representation is based on the microfacet model [TS67, CT82]. Specifically, the normal distribution function (NDF) is approximated using one ASG, and then the BRDF at a specific view slice is obtained through an ASG spherical warping operator. Denote the view direction as o, the reflected direction from p_0 to p_1 as r, the anisotropic BRDF $\rho(r,o)$ is approxiamted by a warped ASG:

$$\rho(r,o) \approx M(r,o)G(r;[x,y,z],[\lambda,\mu]),$$

where M is a smooth function that combines the shadowing term and Fresnel term; G is an ASG; x,y,z are the tangent, bi-tangent, lobe axes, respectively; λ and μ are the bandwidths for x- and y- axes. Those parameters of the ASG G are computed from the NDF function and the view direction o through the ASG warping operator, hence it is also referred to as the warped ASG. For simplicity, we denote the warped ASG $G(r; [x;y; z]; [\lambda;\mu])$ as $G(r)$ for short.

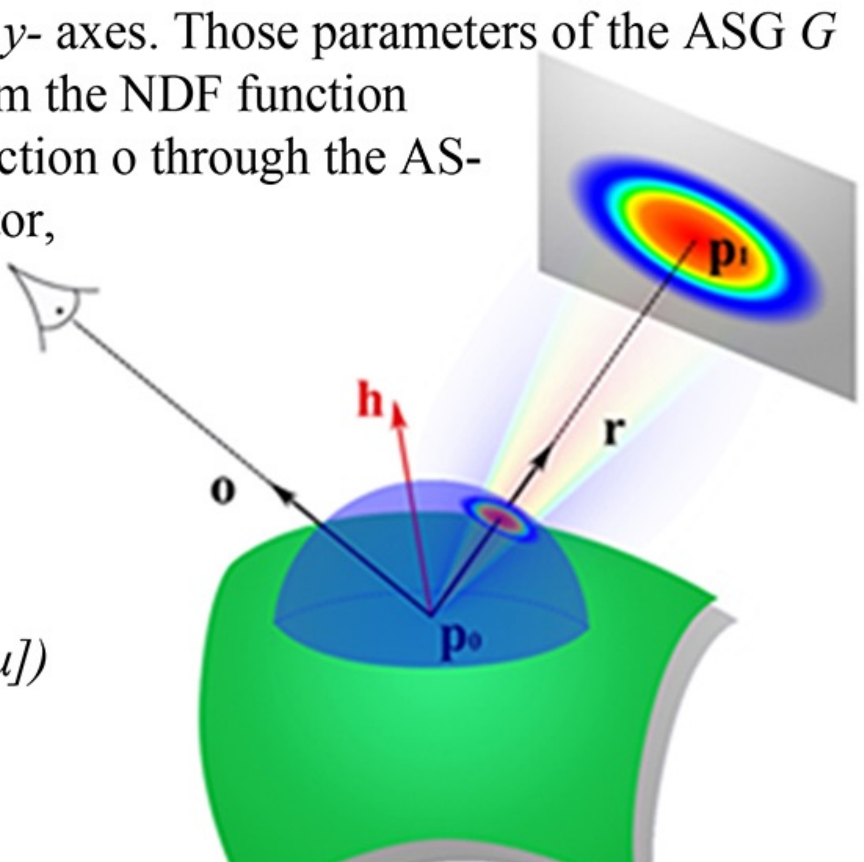

Figure 1. Illustration of spherical warping.

2.Kernel Reconstruction.

We first compute the gradient g of the warped ASG $G(r)$ using this formula:

$$g = \nabla G - (G \cdot r)\, r,$$

after that, we obtain the directions of the minor axis $u_d = g/||g||$, and major axis $v_d = r \times u_d$.

Now we determine the lengths of the minor/major axes. We obtain the axis lengths by constraining relative value changes inside the ellipse within a predefined threshold ε (0.02 in implementation). Along the minor axis, we approximate the value change using first order Taylor expansion at direction r, and the minor axis length l_u satisfies:

$$(\partial G/\partial \mathbf{u}_d) \cdot l_u = \varepsilon \cdot G \Rightarrow l_u = \frac{\varepsilon \cdot G}{\partial G/\partial \mathbf{u}_d},$$

and second order Taylor expansion at direction r to obtain the major length l_v :

$$\frac{1}{2} \cdot \frac{\partial^2 G}{\partial \mathbf{v}_d^2} \cdot l_v = \varepsilon \cdot G \Rightarrow l_v = 2\frac{\varepsilon \cdot G}{\partial^2 G/\partial \mathbf{v}_d^2},$$

we now obtain minor/major axes:

$$u = l_u u_d,\ v = l_v v_d.$$

3.Planar Projection.

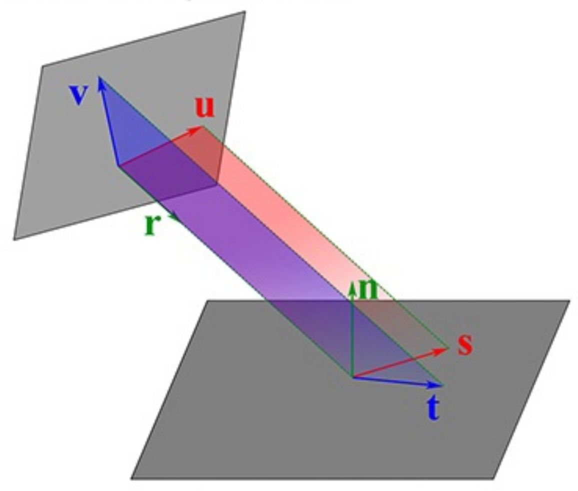

Figure 2. Illustration of planar projection.

As shown in Figure 2, we need to project this elliptical kernel from direction space to tangent plane of the density estimation point p_1, since the density estimation is finally performed there:

$$s = u - (u \cdot n / r \cdot n) \cdot r,$$
$$t = v - (v \cdot n / r \cdot n) \cdot r.$$

4. Density Estimation.

After obtaining the ellipse on the tangent plane of the density estimation point p1, we now explain how to use our kernel for filtering. Specifically, we explain how to compute weight for each photon.

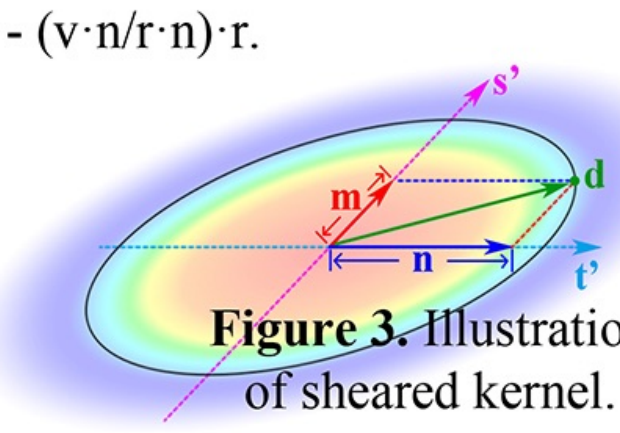

Figure 3. Illustration of sheared kernel.

As shown in Figure 3, the weight used in density estimation is computed as a Gaussian:

$$w(\mathbf{d}) = exp(-a^2 - b^2).$$

a and b can be obtained using following formula:

$$a = \frac{m}{\|\mathbf{s}'\|} = \frac{(\mathbf{s}' \cdot \mathbf{d}) \cdot \|\mathbf{t}'\|^2 - (\mathbf{s}' \cdot \mathbf{t}') \cdot (\mathbf{t}' \cdot \mathbf{d})}{\|\mathbf{s}'\|^2 \|\mathbf{t}'\|^2 - (\mathbf{s}' \cdot \mathbf{t}')^2},$$

$$b = \frac{n}{\|\mathbf{t}'\|} = \frac{(\mathbf{t}' \cdot \mathbf{d}) \cdot \|\mathbf{s}'\|^2 - (\mathbf{s}' \cdot \mathbf{t}') \cdot (\mathbf{s}' \cdot \mathbf{d})}{\|\mathbf{s}'\|^2 \|\mathbf{t}'\|^2 - (\mathbf{s}' \cdot \mathbf{t}')^2}.$$

3. Results

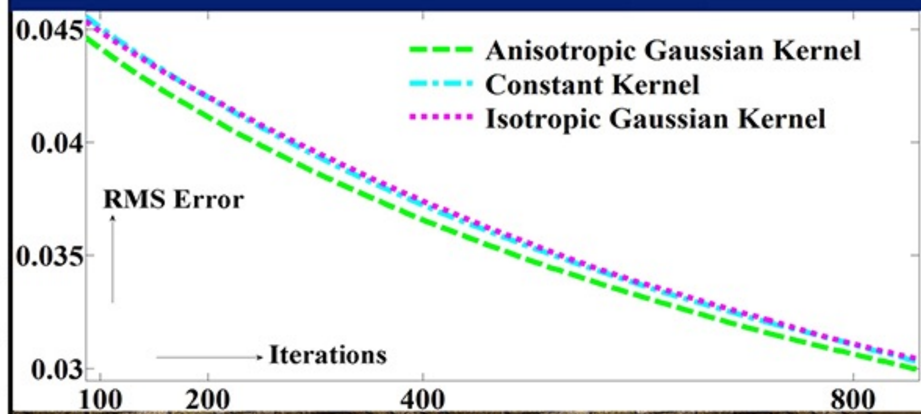

Figure 4. Error plot of MSEs for isotropic kernels and our anisotropic kernel.. Our method consistently produces smaller errors.

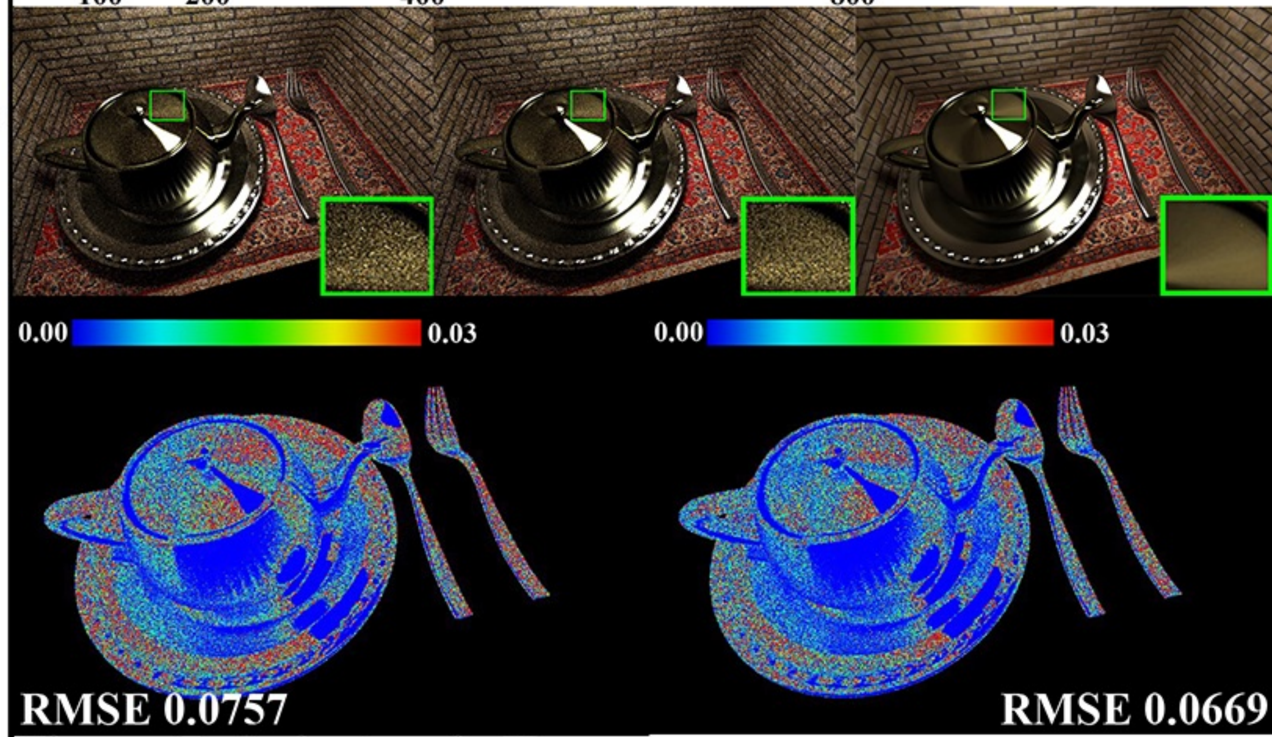

Figure 5. Anisotropic Teapot.

Top from left to right: isotropic kernel, our kernel and reference. Bottom left isotropic, right ours.

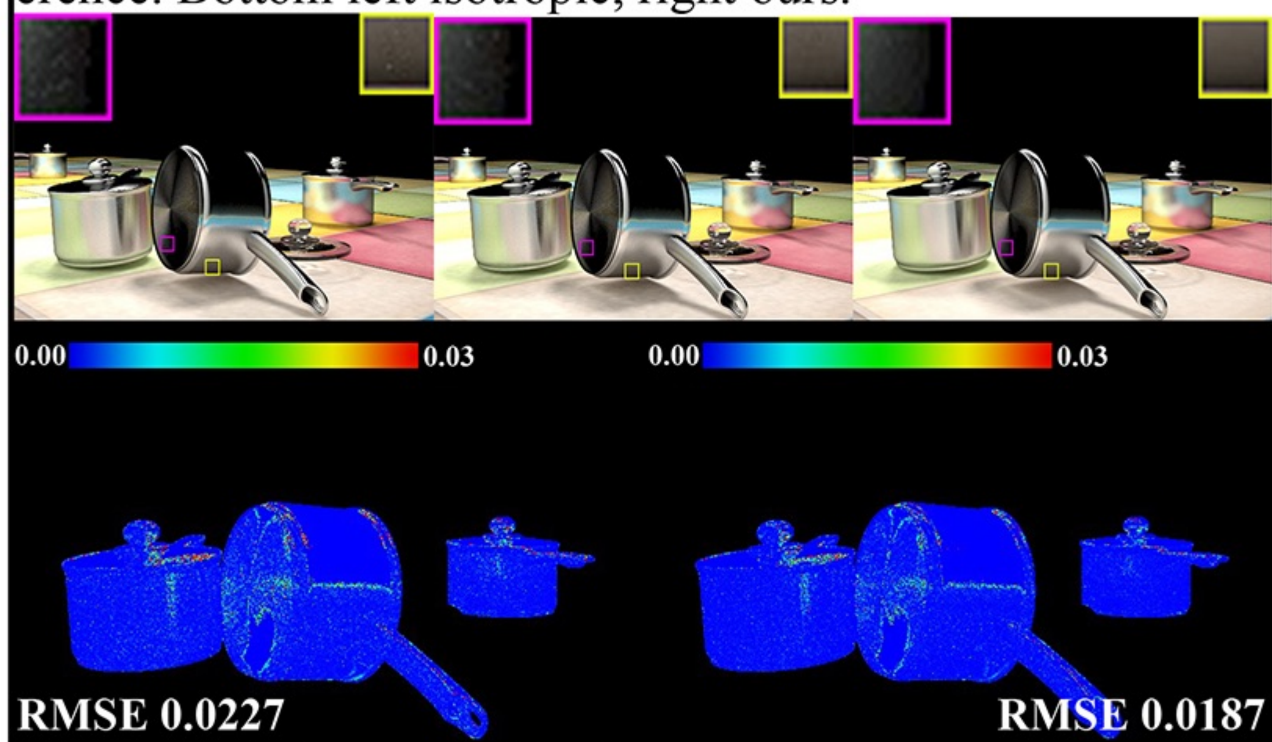

Figure 6. Anisotropic Fry Pan.

Top from left to right: isotropic kernel, our kernel and reference. Bottom left isotropic, right ours.

4. Acknowledgements

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5. References

[XSD*13] K. Xu, W.-L. Sun, Z. Dong, D.-Y. Zhao, R.-D. Wu, and S.-M. Hu. Anisotropic spherical gaussians. ACM Transactions on Graphics (TOG), 32(6):209, 2013.

[CT82] R. L. Cook and K. E. Torrance. A reflectance model for computer graphics. ACM Transactions on Graphics (TOG), 1(1):7–24, 1982.

[TS67] K. E. Torrance and E. M. Sparrow. Theory for off-specular reflection from roughened surfaces. JOSA, 57(9):1105–1112, 1967.